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Tutorial Sheet

Fourier transforms:

Fourier transforms, Fourier sine and Fourier cosine transforms and their properties.

Fourier transforms:

1. Define Fourier Transformation and explain how does it differ from the Fourier series?
2. State and explain Dirichlet's conditions for Fourier Transform. Why are these conditions important.
3. Discuss the properties of the Fourier Transform. Explain important properties such as convolution theorem, and duality.
4. What is the significance of the Fourier Transform in solving differential equations? Explain how it is used to transform and solve partial differential equations (PDEs) and ordinary differential equations (ODEs) in applied mathematics and physics.
5. Differentiate between Fourier Sine Series and Fourier Cosine Series. When is each used?
6. State the general formula for the Fourier series of $f(x)$ in $(-L, L)$, giving general expressions for the coefficients of the series.
7. Find the Fourier transform of the following function

$$(i) F(x) = e^{-|x|} \quad (ii) F(x) = \frac{1}{2\varepsilon}, |x| \leq \varepsilon \quad \text{Ans: } (i) \frac{2}{1+p^2} \quad (ii) \frac{\sin p\varepsilon}{p\varepsilon}$$

8. Find the Fourier transform of the given function $F(x) = \frac{\sin ax}{x}, a > 0$.

Ans: Case 1 : $f(p) = \pi$ and Case 2: $f(p) = 0$

9. Find the complex Fourier transform of dirac delta function $\delta(x-a)$. **Ans:** e^{ipa}

10. Find the inverse Fourier transform of $f(p) = e^{-|p|y}$. **Ans:** $\frac{y}{\pi(y^2 + x^2)}$

11. Find Fourier sine transformation of $\frac{e^{-ax}}{x}, a > 0$ hence find Fourier sine

$$\text{transformation of } \frac{1}{x}. \quad \text{Ans: } \frac{\pi}{2}$$

12. Find the Fourier cosine transformation of $\frac{1}{1+x^2}$ and hence find Fourier sine

transformation of $\frac{x}{1+x^2}$.

Ans: $\frac{\pi}{2} e^{-p}$

13. Find the Fourier cosine transformation of e^{-x^2} .

Ans: $\frac{\sqrt{\pi}}{2} e^{-\frac{p^2}{4}}$

14. Solve the integral equations: $\int_0^\infty f(x) \cos \lambda x dx = e^{-\lambda}$. **Ans:** $F(x) =$

$\frac{2}{\pi x} (1 + \cos x - 2 \cos 2x)$

15. Find $F(x)$ if its Fourier sine transformation is $\frac{\pi}{2}$.

Ans: $\frac{1}{x}$

16. Find Fourier sine and cosine transformation of $\frac{1}{\sqrt{x}}$.

Ans: $\sqrt{\frac{\pi}{2p}}, \sqrt{\frac{\pi}{2p}}$

17. Find Fourier finite sine transformation of $F(x) = 1 - \frac{x}{\pi}$.

Ans: $\frac{1}{p^2 - k^2}, k \neq 0, 1, 2, \dots$

18. Prove that: $\int_0^\infty \left(\frac{\sin x}{x} \right)^4 dx = \frac{\pi}{3}$

19. Using Parseval's identity, show that $\int_0^\infty \frac{x^2}{(a^2 + x^2)(b^2 + x^2)} dx = \frac{\pi}{2(a+b)}$.

20. Find Fourier finite cosine transformation of $\sin nx, n \in I$ **Ans:** $\frac{2n}{n^2 - p^2}; (n-p)$ is odd and 0 if even

21. State and prove the Convolution Theorem.

22. Find the Fourier series for $f(x) = x$ on the interval $[-\pi, \pi]$. **Ans:** $f(x) = \sum (2/n) (-1)^{n+1} \sin(nx), n = 1, 3, 5, \dots$

23. Find the Fourier series for $f(x) = \sin(x)$ on the interval $[-\pi, \pi]$

Ans: $f(x) = \sin(x)$, This result shows that $\sin(x)$ is already in its simplest Fourier series form.

24. Find the Fourier Series to represent $f(x) = |x|$ in the interval $-\pi < x < \pi$

Ans: $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$

25. Find the Fourier series of $f(x) = 2x, -\pi < x < \pi$, **Ans:** $\frac{4}{n} (-1)^{n+1}$